1. $\alpha(t) = (\sin t, \cos t)$

2. Let $d(t)$ be distance from the origin to $\alpha(t)$, then $d(t) = |\alpha(t)|$

$d(t)^2 = \langle \alpha(t), \alpha(t) \rangle$

Since $d(t) \neq 0$ for all $t \in I$, we differentiate w.r.t. t

$2d(t)d'(t) = 2\langle \alpha'(t), \alpha(t) \rangle$

Since $\alpha(t_0)$ is closest to the origin, $d'(t_0) = 0$

$\langle \alpha'(t_0), \alpha(t_0) \rangle = d(t_0)d'(t_0) = 0$

3. Let $\alpha(t) = (x(t), y(t), z(t))$

Since $\alpha''(t) = 0$, $x''(t) = y''(t) = z''(t) = 0$

Then $x(t) = u_1 t + v_1$

$y(t) = u_2 t + v_2$ $\quad u_1, u_2, u_3, v_1, v_2, v_3$ are constant

$z(t) = u_3 t + v_3$

4. $\frac{d}{dt}\langle \alpha(t), v \rangle = \langle \alpha'(t), v \rangle = 0$

Then $\langle \alpha(t), v \rangle = c$ for some constant c

Since $\langle \alpha(0), v \rangle = 0$, $\langle \alpha(t), v \rangle = 0$ for all $t \in I$

$\alpha(t)$ lies on the plane that has normal vector v and passes through the origin

$\alpha(t)$ is a plane curve

5. $\alpha : (-1, +\infty) \longrightarrow \mathbb{R}^2$, $\alpha(t) = \left(\frac{3t}{1+t^3}, \frac{3t^2}{1+t^3}\right)$

Suppose $\alpha(t_1) = \alpha(t_2)$

Then $\frac{3t_1}{1+t_1^3} = \frac{3t_2}{1+t_2^3}$ ——— ①

$\frac{3t_1}{1+t_1^3} \cdot t_1 = \frac{3t_1^2}{1+t_1^3} = \frac{3t_2^2}{1+t_2^3} = \frac{3t_2}{1+t_2^3} \cdot t_2$ ——— ②

By ① & ②, $t_1 = t_2$. Hence α is injective.

Consider the mapping $\alpha : (-1, +\infty) \longrightarrow$ Image of α

α^{-1} is not cts at $(0,0)$

$\alpha(0) = (0,0)$ $\alpha^{-1}((0,0)) = 0$ $\lim_{t \to \infty} \alpha(t) = (0,0)$

i.e. For any $r > 0$, $\exists$ M such that

$\alpha(t) \in B_r$ $\forall t > M$

$B_r = \{ (x,y) \in \mathbb{R}^2 \mid x^2 + y^2 < r \}$

Then let $\varepsilon > 0$, for any open ball containing $(0,0)$

$\alpha^{-1}(B_r) \not\subset (-\varepsilon, \varepsilon)$